Name: __________________
Student Number: __________________

Exam on "Solid State Physics"

WBPH068-05.2023-2024.1

Content: 5 pages (including this cover page)

Friday 25 Jan 2024; 15:00 - 17:00

Exam drafted by (name first examiner): Jianting Ye

Exam reviewed by (name second examiner): Graeme Blake

For administrative purposes; do NOT fill the table

	Maximum points	Points scored
Question 1	26	
Question 2	24	
Question 3	24+5	
Question 4	24+5	
Total	100	

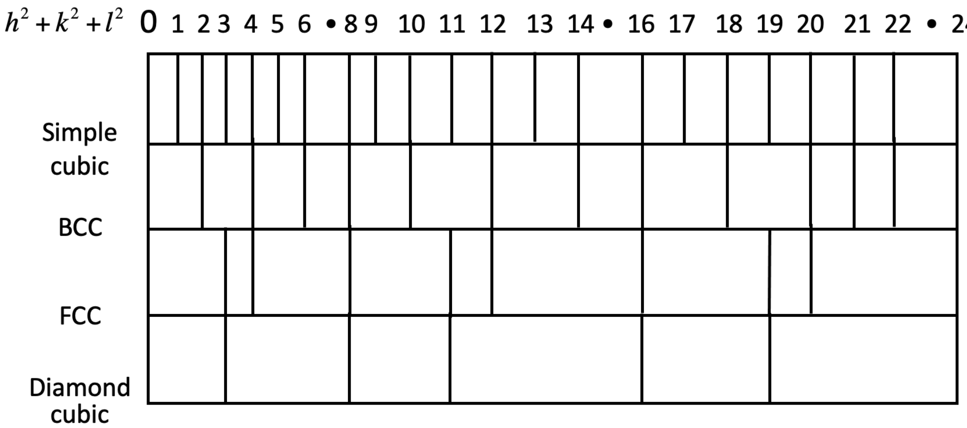
Final mark: __________

Question 1: Crystal structure and X-ray scattering

Note that a rough hand sketch is sufficient, don't spend your time making good-looking drawings!

1) (10p) For the crystal types in the cubic system, considering a conventional cubic cell of lattice constant a, draw the simple cubic (SC), body-centered cubic (BCC), and face-centered cubic (FCC) lattices. What are the respective sizes of their primitive cells?
2) (6p) When observing fruit arrangements below in an Asian market, you might notice that fruits, such as oranges, are often stacked in tetrahedrons to attract customers. For orange piles formed by close packing, determine the type of packing (AB or ABC) and the crystal structure. Assuming uniform sizes for each orange, calculate the volume fraction of the packing.

3) (10p) Explain (qualitatively is sufficient) the evolution of x-ray diffraction lines in the cubic system shown below. Why do the diffraction lines decrease as the complexity of the basis increases (or symmetry reduces)? The structure factor is given by $S_G = \sum_j f_j \exp(-i\vec{G}\cdot\vec{r}_j)$, where f_j represents the atomic scattering factor and the sum is taken over all atoms in the unit cell.



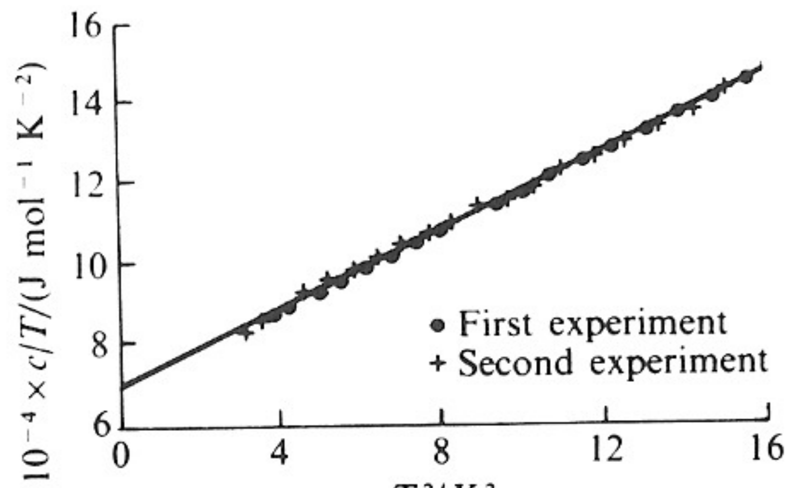
Question 2: Phonons and thermal properties.

Consider a linear chain of N atoms, all with mass M and force constant C.

1) (10p) Calculate and sketch the dispersion relation in the first Brillouin zone for the lattice vibration of the chain, where the interatomic distance is a.
2) (10p) Sketch the dispersion relationship if the mass or force constant changes, *i.e.*, if the monoatomic chain becomes a diatomic chain. How does the $\omega(k)$ relationship evolve with respect to the answer in question 1?
3) (6p) Suppose the pairwise potential between the 1D monoatomic chain is the so-called Lennard–Jones potential, $V(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$. Estimate the lattice constant a, binding energy E, and force constant C.

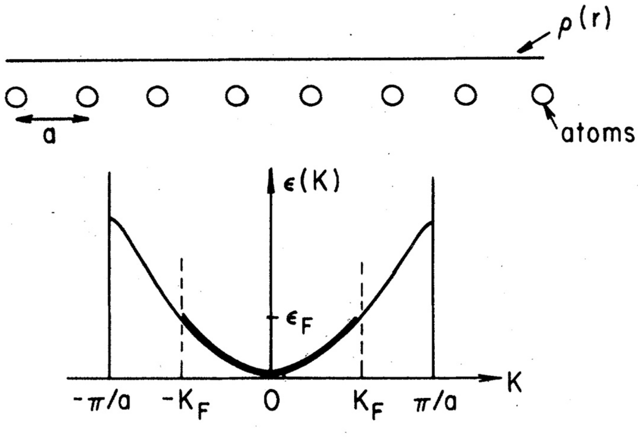
Question 3: Free electrons in metals

1) (10p) Consider a 1D free electron gas formed in a mono-valent metal with a lattice constant a and a total length L. Calculate the Fermi energy E_F and the electron density of states $D(E)$ (hint: be aware of double counting in the states).
2) If the transport of electrons is characterized by the Drude model, demonstrates that assuming a single parameter τ, the mean scattering time is sufficient to describe Ohm's law.
3) (8p) At low temperatures far below the Debye and Fermi temperatures (T_D and T_F, respectively), repeated measurements of specific heat in a 3D metal consistently show a linear relationship between $\frac{c}{T}$ and T^2. Find the contributions to the specific heat from electrons and phonons from the measurement result shown below.

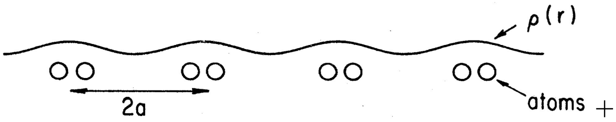


4) (Bonus 5p) In 1955, R. Peierls showed that a one-dimensional electron gas of a mono-valent chain, subject to a crystal lattice, is not stable at low temperatures. It forms a new state, called the charge density wave (CDW) state. Explain the consequence of forming such a state (in the $E(k)$ relationship shown below). And argue why this state is preferred.

- Before CDW transition

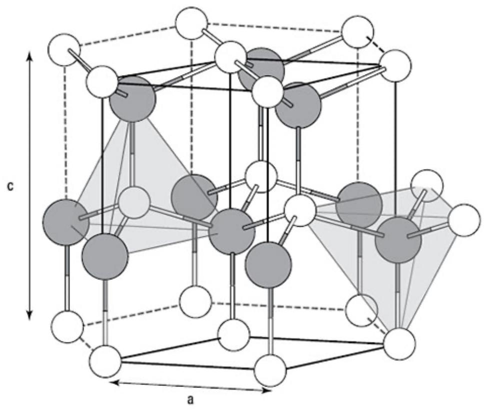


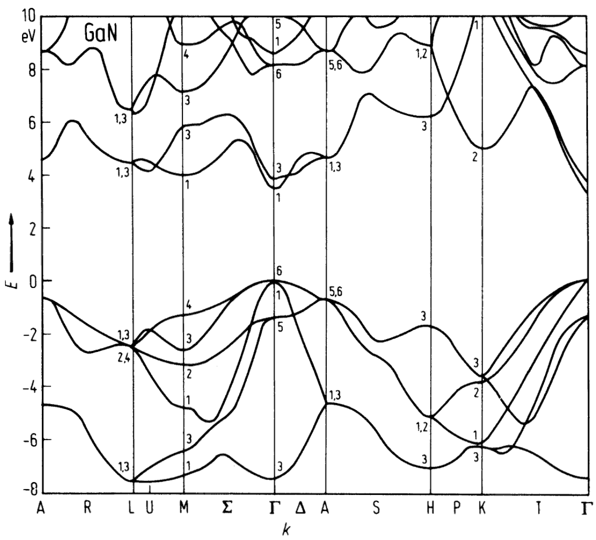
- After CDW transition



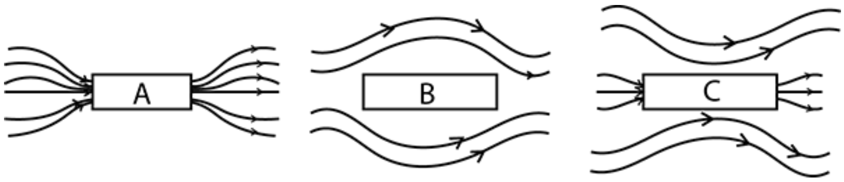
Question 4: Semiconductor and superconductor

1) (10p) GaN (Ga is gray) is a crucial wide-band gap semiconductor used in white-colored LEDs. The discoverers were awarded the Nobel Prize in Physics in 2014. GaN crystallizes in the Wurtzite-type structure (below), which is the other main family of semiconductors besides the Zinc-blend structure.
   Determine the unit cell and basis of GaN from the crystal structure. Find the energy and momenta conservation in the optical absorption process of GaN.
   Sketch the optical absorption intensity versus photon energy, considering that the gap size of GaN is roughly 3.2 eV.



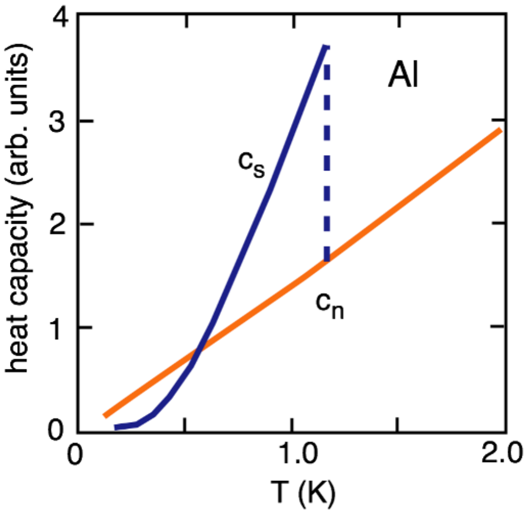


2) (6p) Based on the change of magnetic field shown below, determine for cases A, B, and C which one is diamagnetic, paramagnetic, and ferromagnetic, respectively. Also, specify the magnitude and/or sign of χ in the magnetization.



3) (8p) Consider a metal that demonstrates superconducting properties at a critical temperature T_c. What is the Hallmark change of physical properties that characterize the superconducting transition?

4) (Bonus 3p) In addition to the Hallmarks discussed in the above questions, further experimental details are presented below by measuring the heat capacity of a superconducting state (c_s) and normal state (c_n, orange). The normal state is obtained by applying a large magnetic field that destroys the superconducting state. What insights can we gain from the result regarding the formation of Cooper pairs by pairing two $s = \pm\frac{1}{2}$ electrons?



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